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EMPHASIZING THINKING STRATEGIES IN BASIC FACT INSTRUCTION

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Research and writings by mathematics educators emphasize the importance of thinking strategies in facilitating the teaching and learning of basic facts. Thiele (1938) and Swenson (1949) found convincing evidence that teaching children different thinking strategies helped the learning and retention of the basic addition facts as well as the transfer of that knowledge to other problems. Rathmell (1978), working with young primary school children, found a positive correlation between high achievement scores on addition fact tests and the use of more mature strategies for finding addition sums. Many of the strategies used by the students in his study paralleled those singled out in the Thiele and Swenson studies, such as counting on, using doubles, thinking one more or one less than a known fact, using ten, and recognizing the commutative idea. Interviews conducted by Myers and Thornton (1977) with regular and special class students supported Rathmell's findings for all operations. Children who were successful with the facts tended to discover and use relationships among facts and other techniques as a help for memorizing them, but left to their own resources, slower learners and LD students did not.

Rathmell (1978) proposed that if children are to learn more mature and efficient methods of solving basic facts, these methods must be explicitly taught. Ashlock (1971) and Hall and Trafton (1974) have suggested that helping children develop thinking strategies is a necessary middle step between concept development for the operations and drill for mastery of facts. Unfortunately, such development is not always common practice in mathematics classrooms. School visitations and a review of current mathematics texts by the investigator reveal that the focus on thinking strategies is usually narrow and limited. Although the use of mathematical properties (particularly the commutative and distributive principles) is stressed, many other recommendations from the literature regarding the relating of facts or the sequencing of facts for instruction receive little consistent attention. As Jerman (1970) pointed out on the basis of his research on children's strategies with multiplication facts, "If students really continue to use strategies first learned, . . . [then] perhaps the combinations should be introduced in terms of their relative difficulty, which involves both the magnitude and the structure of each combination" (p. 127).

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The Study

The purpose was to explore the effectiveness of resequencing the instruction of basic facts in order to emphasize thinking strategies for remembering the facts. Two parallel investigations, at the second- and fourth-grade levels, were conducted. Specifically, at each grade level the following questions were examined:

1. Based on total test scores
   a. Were there any differences in the mean performances on posttests and retention tests between groups receiving experimental (E) and traditional (T) instruction on basic facts?
   b. Within groups (E and T), were there any differences in mean performance on the pretest, posttest, and retention test?
2. Based on harder facts only
   a. Were there any differences between groups in the mean number of harder facts answered correctly by students on posttests and retention tests?
   b. Within groups, were there any differences in the mean number of harder facts answered correctly on the pretest, posttest, and retention test?
3. Were there differences in the thinking strategies actually adopted—
   a. By students in the E and T groups?
   b. By high- and low-achieving students?

Subjects

The population for the study consisted of second- and fourth-grade pupils in two elementary schools near Illinois State University. Pupils in the two schools had similar socioeconomic backgrounds. For the sample it was necessary to pool intact groups from multilevel classes to form an experimental group (E2, $n = 25$) and a traditional group (T2, $n = 22$) at the second-grade level, and at the fourth-grade level, an experimental group (E4, $n = 23$) and a traditional group (T4, $n = 20$). The mean IQs of students in the sample were as follows: $\overline{E2}$, 110.8; $\overline{T2}$, 110.4; $\overline{E4}$, 107.2; $\overline{T4}$, 108.4. Random selection of pupils for groups was not possible, but random assignment of treatments to groups was made.

Teachers for the T and E Groups

Since it was not feasible to assign the same teacher to all groups, it was impossible to control totally the teacher variable during the study. Although each teacher involved in the study was experienced (each had taught at least six years in school classrooms), differences in teaching style, personality, individual creativity in motivating students to learn, and other related aspects that can influence learning and achievement by pupils were not controlled. Prior to and during the study, however, attempts were made

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to minimize differences among teachers influencing T and E group instruction.

Teachers agreed (a) on the testing procedures and on the time constraints, the nature and sequence of instruction; (b) that no formal work on basic facts should occur outside the time prescribed for instruction of facts; (c) to tell parents at the first conference that they could help students on facts if they wished, since this was a customary practice in both schools (parents were not, however, to be urged to make a special "all-out effort" at this time); (d) to let absences randomize themselves out, rather than to set aside special make-up sessions for students (only prolonged absences would have been reported, although none actually occurred); (e) to spend mathematics time outside the assigned period of instruction for basic facts on content topics such as geometry, time, or metric measurement. Teachers met with the investigator several times both before and during the study to check whether the guidelines were understood and followed by all, and teachers communicated among each other and with the investigator when questions arose.

Procedure

A timeline outlining the study is presented in Figure 1. Pretests were administered during the second week of school in the fall. E2 and T2 students were given three-minute written timings on both addition and subtraction basic facts. E4 and T4 students took similar timings on multiplication and division basic facts. The addition, subtraction, and multiplication tests each contained 98 fact problems written in vertical form (e.g., $\begin{array}{r}6\\ \underline{\times 3}\end{array}$).

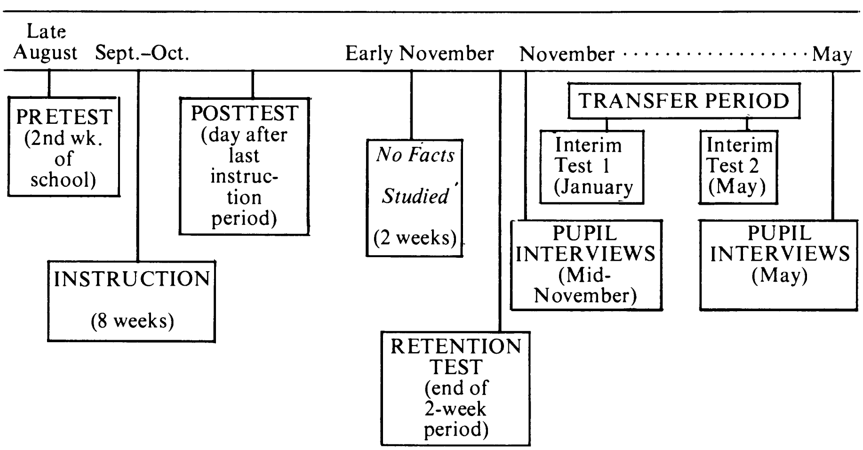

Figure 1. Timeline of Study Throughout Academic Year.

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The division test consisted of 91 fact problems of the form $3\overline{)18}$. Essentially all basic facts for each operation were presented in random order on each test. One fact was added to the division test and two easier facts were deleted from the others to even out rows.

Eight weeks of instruction followed the pretesting. At each grade level instruction occurred for 20 minutes a day on three consecutive days during each of these weeks. T-group instruction at each level was based on the sequence, teaching suggestions, and assignment sections of the adopted mathematics text, together with supplementary teacher-directed drills (like "Climb the addition/multiplication ladder, fast!" or "Move with the (multiplication) record," and so on) during the class periods. The T2 instruction first reviewed sums, then differences to 10 before studying sums, and then related differences to 18. T4 instruction first focused, in order, on the easier products (2s, 3s, 4s, and 5s) and then turned to the harder multiplication facts (6s, 7s, 8s, 9s). Related division facts were also studied, though less time was spent on these than on multiplication within the eight-week period.

E group instruction at each level was based on the special sequence, teaching (developmental and drill) suggestions, and assignment sections of the special booklets (prepublication copies of *Look into the facts* [two booklets: *Addition and subtraction, multiplication and division*, with accompanying manuals], now published by Creative Publications) prepared by the investigator to emphasize the relating of harder facts to easier, known ones. E2 instruction first required the mastery of doubles, the prerequisite to studying related facts—doubles + 1 like 5 + 6 and 7 + 6, then "sharing numbers" like 6 + 8 and 7 + 5 (sharing numbers, which differ by two, can be "turned into" doubles, e.g., 7 + 5 = 6 + 6 [we've "shared" by taking "1" from the 7 and giving it to the 5]. Some children relate 5 + 7 to a double by thinking "5 + 5 and 2 more."). Addition 9s were then presented (one less than adding 10 to a number); and finally the 10 remaining facts and their commutatives were studied: 2 + (5, 6, 7, 8); 3 + (6, 7, 8); 4 + (7, 8); and 5 + 8. Ideas like counting on and using 10 formed the basis of the strategies encouraged for remembering these last facts. Those who mastered the addition facts in the eight-week time period turned to subtraction facts and were prompted to "think of the addition fact" to find their answers. E4 instruction studied, in order, multiplication of 2s, 5s, 9s, and squares before turning to the "last 10" facts (3 × (4, 6, 7, 8); 4 × (6, 7, 8); 6 × (7, 8) and 7 × 8) and division. Patterns, relationships, and other techniques (e.g., twice as much, adding on, subtracting from, finger multiplication for 9s, commutativity) were emphasized. For division, students were encouraged to "think of the multiplication fact." In the E groups there was heavy emphasis on helping children to organize their thinking, to create their own or adopt suggested strategies for remembering the facts *prior* to drill over any given segment. Because of the need to master easier facts (so that they could be subsequently used to help with harder ones), there tended to be more subgrouping in the E than in the T groups.

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Posttests were administered to students in each group on the day following the last instruction period. Then, for two weeks *no facts* were studied by the students so that a retention test could be given. The pretest, posttest, and retention test for each operation were identical. Each test (for any given operation) had a prescribed three-minute time limit. At the beginning of each testing session children were encouraged to skip any problem they didn't know right away, to do the easy ones first and come back later.

The top and lower thirds of each group (identified by the average of posttest and retention test scores) were interviewed by the investigator immediately after the study to gain information concerning strategies actually adopted by students. A series of follow-up interviews (the subjects of which were chosen on the basis of the average scores of the January and May tests, which were the same as the tests described above) were conducted during the first week of May with the top and lower thirds of each group. In each case a student was first presented, one by one, the following five facts: $2 + 6$, $4 + 7$, $6 + 7$, $9 + 4$, $8 + 6$ (second-grade subjects); or 3×8, 5×7, 4×6, 7×9, 6×8 (fourth-grade subjects). Children were asked to think back to when they were first learning these facts and to tell any tricks, shortcuts, or other easy ways they used for remembering them. They were then shown a blank addition (or multiplication) test form and were asked whether they knew any easy ways for remembering other facts on the sheet.

In the interim between the retention test and the May interviews, regular class teachers for the subjects involved helped them with the facts not mastered during the study and resumed their regular mathematics program. The adopted text, which was the same for both schools, periodically presented reviews of the facts throughout the school year.

Data Analysis and Results

The analysis was partitioned into three sections to correspond to the three major questions of the study. The first two sections of this analysis assessed differences within and between groups (a) in mean performance on the pretest, posttest, and retention tests; and (b) on the mean number of harder facts answered correctly on each of these tests. Since random assignment of pupils to groups (E and T) was not made, the unit of analysis was groups rather than individual students at each grade level. For the analyses a general 2×3 design with repeated measures on the second factor was used to test, for each operation, main effects and interaction effects. Scheffe Tests for Multiple Comparisons (Winer, 1971) and t tests were subsequently carried out to locate significant differences.

Group Differences: Total Test Performance

Group means on the three dependent measures (total test score on pretests, posttests, and retention tests) and corresponding standard deviations for each group are presented in Table 1. With the exception of the T2 group, heterogeneity within groups tended to increase over time. As would be

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expected, mean group performance, which was evaluated on the basis of total test scores, also improved during the study. The one notable exception of the T2 group in subtraction is discussed later in this report.

Table 1
Means and Standard Deviations for Each Operation on Total Test Scores

	Group	Pretest	Posttest	Retention Test
Addition	E2	23.16 (7.67)[a]	61.64 (17.26)	65.20 (18.93)
	T2	32.32 (13.01)	39.36 (10.96)	36.59 (10.21)
Subtraction	E2	16.56 (7.55)	41.56 (16.02)	44.00 (17.68)
	T2	27.86 (14.50)	20.59 (9.94)	20.59 (10.60)
Multiplication	E4	19.22 (14.90)	70.22 (16.55)	71.22 (22.11)
	T4	21.20 (11.70)	51.10 (15.12)	55.60 (14.73)
Division	E4	9.91 (11.26)	58.87 (24.20)	58.78 (26.08)
	T4	16.30 (11.38)	43.60 (15.90)	43.40 (15.02)

[a] Figures in parentheses are the standard deviations.

The separate two-way ANOVAs for each operation are summarized in Tables 2 and 3. The group × total test interactions were significant ($p < .001$) for each operation.

The group × total test-interaction graphs are presented in Figure 2. In each graph the marked effect of E treatment over time is demonstrated. With the exception of subtraction, the T2 group performed better on the pretest than on either the posttest or retention test.

The Scheffe analysis, which was subsequently used to examine pairwise differences within groups on total test performance, is summarized in Table 4. Significant pretest–posttest gains occurred within both E and T groups for all operations except subtraction, where the loss for the second-grade traditional group was significant. The pretest–retention test gain was also significant across groups except in the T2 group, where the gain was nonsignificant for addition, and a significant loss was noted for subtraction. In no case was the posttest–retention test difference significant.

Two-tailed t tests were used in the post hoc analysis to locate specific differences between groups. The results are summarized in Table 5. Even though pretest means were significantly lower for the E2 than for the T2 groups ($p < .01$), both posttest and retention-test differences favored the E2 group for addition and for subtraction ($p < .001$). At the fourth-grade level pretest differences were not significant, and posttreatment tests showed significant gains at the .05 level or beyond favoring the E4 group.

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Table 2
Grade 2: Summary of Repeated Measures ANOVA
Mean Group Performance, Total Score: Pretest, Posttest, Retention Test

Source of variation	Addition		F
	df	MS	
Between subjects	46	552.04	
Groups E and T (G)	1	6791.78	16.42**
$Error_b$	45	413.38	
Within subjects	94	379.44	
Total Test Performance (T_o)	2	9076.23	102.88**
$G \times T_o$	2	4787.34	54.26**
$Error_w$	90	88.22	
Total	140	436.15	
	Subtraction		
Between subjects	46	508.01	
Groups E and T (G)	1	4267.09	10.05*
$Error_b$	45	424.47	
Within subjects	94	192.94	
Total Test Performance (T_o)	2	1761.07	27.19**
$G \times T_o$	2	4393.09	67.84**
$Error_w$	90	64.76	
Total	140	296.46	

** $p < .001$
* $p < .01$

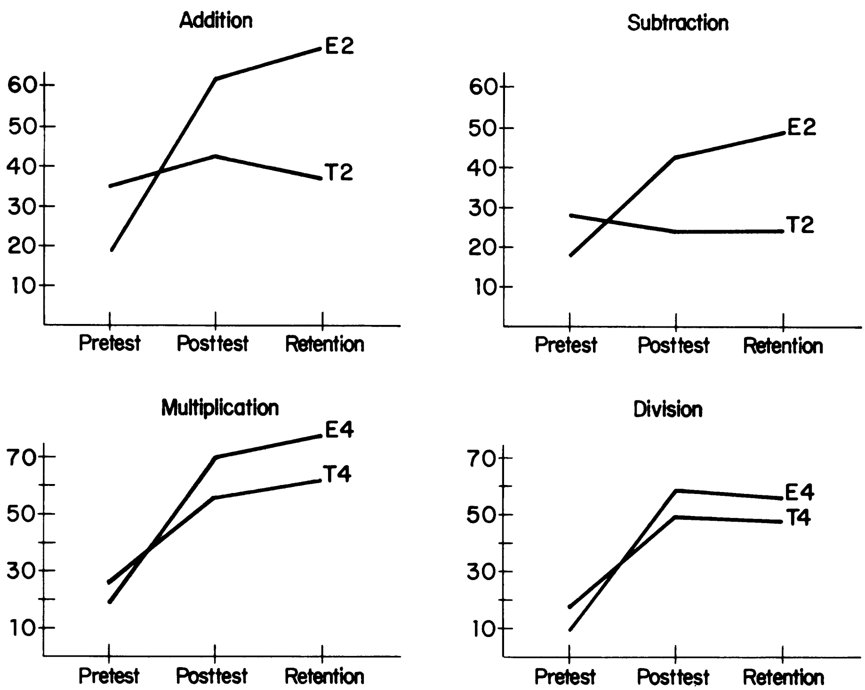

Figure 2. Group × Total Test Interaction Graphs

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Table 3
Grade 4: Summary of Repeated Measures ANOVA
Mean Group Performance, Total Test Score: Pre-, Post-, Retention

Source of variation	*df*	MS	F
	Multiplication		
Between subjects	42	687.43	
Groups E and T (G)	1	3825.15	6.26*
$Error_b$	41	610.90	
Within subjects	86	743.28	
Total Test Performance (T_o)	2	25963.82	229.95**
G × T_o	2	1367.91	12.16**
$Error_w$	82	112.91	
Total	128	724.95	
	Division		
Between subjects	42	890.72	
Groups E and T (G)	1	2099.60	2.44
$Error_b$	41	861.23	
Within subjects	86	644.50	
Total Test Peformance (T_o)	2	21593.71	199.46**
G × T_o	2	1681.20	15.53**
$Error_w$	82	108.26	
Total	128	725.29	

** $p < .001$
* $p < .01$

Table 4
Scheffe Post Hoc Analysis: Pairwise Comparisons
Within Groups on the Total Test Performance: Pre-, Post-, and Retention

	Comparison	Difference of Means[a]	Comparison	Difference of Means
	Group E2		Group T2	
Addition	$T_2 - T_1$[b]	38.4**	$T_2 - T_1$	7.1*
	$T_3 - T_1$	42.0**	$T_3 - T_1$	4.3
	$T_3 - T_2$	3.6	$T_3 - T_2$	−2.8
Subtraction	$T_2 - T_1$	25.0**	$T_2 - T_1$	−7.3*
	$T_3 - T_1$	27.4**	$T_3 - T_1$	−7.3*
	$T_3 - T_2$	2.4	$T_3 - T_2$	0.0
	Group E4		Group T4	
Multiplication	$T_2 - T_1$	51.0**	$T_2 - T_1$	29.9**
	$T_3 - T_1$	52.0**	$T_3 - T_1$	34.4**
	$T_3 - T_2$	1.0	$T_3 - T_2$	4.5
Division	$T_2 - T_1$	49.0**	$T_2 - T_1$	27.3**
	$T_3 - T_1$	48.9**	$T_3 - T_1$	27.1**
	$T_3 - T_2$	−0.1	$T_3 - T_2$	−0.2

[a] Critical values for E2(α = .001): 10.32 (addition); 8.85 (subtraction);
T2(α = .05): 7.05 (addition); 6.05 (subtraction);
E4(α = .001): 12.20 (multiplication); 11.93 (division);
T4(α = .001): 13.10 (multiplication); 12.83 (division).

[b] Total test performance on Pretest (T_1), Posttest (T_2), Retention Test (T_3).
** $p < .001$
* $p < .05$

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Table 5
Summary of t-test Analysis Between Groups E and T
By Grade on Total Test Performance

	t values[a]			
	Grade 2 ($df = 45$)		Grade 4 ($df = 41$)	
	Addition	Subtraction	Multiplication	Division
Pretest	−2.92**	−3.33**	−0.47	−1.80
Posttest	5.09***	5.19***	3.84***	2.36*
Retention Test	6.19***	5.30***	2.62*	2.27*

[a] E − T difference is reflected in each t value.
*** $p < .001$
** $p < .01$
* $p < .05$

Group Differences: Harder Facts

The dependent variable in the second major section of the analysis was the mean number of "harder" facts answered correctly on the pretests, posttests, and retention tests. Since the eight-week instructional period focused principally on the addition and multiplication facts, only tests for these operations were analyzed.

For the purpose of the study a harder fact was one in which both addends (factors) were greater than 3 and at least one was greater than 6. The addition test contained 40 harder facts, the multiplication test 49 (out of 98 total on each test). These were distributed rather evenly throughout the test. Table 6 presents means and corresponding standard deviations for each group on harder fact performance. Pretest means are quite similar. Differences between the two groups are more notable in the post and retention assessments.

Table 6
Means and Standard Deviations
Harder Facts Answered Correctly on Pretest, Posttest, Retention Tests

	Group	Pretest	Posttest	Retention Test
Addition	E2	5.52 (4.66)[a]	17.24 (11.33)	19.32 (11.51)
	T2	5.14 (4.12)	3.91 (3.78)	5.87 (5.68)
Multiplication	E4	5.09 (4.94)	29.74 (13.06)	29.91 (13.75)
	T4	5.80 (3.68)	15.00 (8.00)	15.50 (7.74)

[a] Figures in parentheses are the standard deviations.

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The ANOVA summary for each operation is presented in Table 7. In each case the main effects and the group × harder fact interactions were significant ($p < .001$).

Table 7
Summary of Repeated Measures ANOVA
Mean Group Performance for Harder Facts: Pre-, Post-, Retention Tests

	df	MS	F
	Addition		
Between subjects	46	204.09	
Groups E and T (G)	1	2879.73	19.91*
$Error_b$	45	144.63	
Within subjects	94	50.59	
Harder facts (H)	2	745.04	34.48*
G × H	2	660.28	30.56*
$Error_w$	90	21.61	
Total	140	101.03	
	Multiplication		
Between subjects	42	276.61	
Groups E and T (G)	1	2884.04	13.54*
$Error_b$	41	213.01	
Within subjects	86	156.10	
Harder facts (H)	2	4455.12	128.33*
G × H	2	833.84	24.02*
$Error_w$	82	34.72	
Total	128	195.64	

* $p < .001$

Figure 3 presents the interaction graphs for each operation. An examination of the graphs reveals quite different patterns of two-way interactions. The number of correct responses to harder facts consistently increased for the E groups and for the T4 group. The unexpected T2 performance deviated from this pattern.

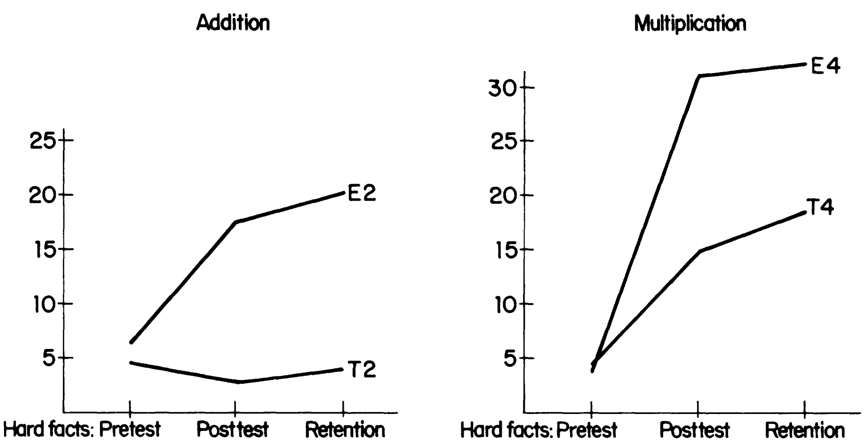

Figure 3. Group × Harder Fact Interaction Graphs

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Pairwise differences *within* groups on performance with harder facts was subsequently examined with a Scheffe analysis. Table 8 summarizes the results of this investigation. After treatment, significant increases occurred in the number of harder addition (multiplication) facts answered correctly by students in all but the T2 group. No significant gains were indicated by the posttest–retention test differences.

Table 8
Scheffe Post Hoc Analysis: Pairwise Comparisons
Within Groups on Harder Fact Performance

	Difference of Means[a]			
	Addition		Multiplication	
Comparison	E2	T2	E4	T4
$H_2 - H_1$[b]	11.72**	−1.23	24.65**	9.20*
$H_3 - H_1$	13.80**	0.73	24.82**	9.70*
$H_3 - H_2$	2.08	1.96	0.17	0.50

[a] Critical values for E2(α = .001): 5.08; T2(α = .05): 3.49;
E4(α = .001): 11.78; T4(α = .05): 8.68.
[b] Harder Facts on Pretest (H_1), Posttest (H_2), Retention (H_3).
** $p < .001$
* $p < .05$

To investigate specific differences between the groups on harder fact performance, two-tailed *t* tests were carried out. The summary data of Table 9 indicates that even though pretest differences for both grades were non-significant, those for the posttests and retention tests were significant ($p <$ .001), favoring the E groups.

Table 9
Summary of *t*-test Analysis Between Groups E and T
By Grade on Hard Fact Performance

	t values[a]	
	Grade 2 (df = 45) (Addition)	Grade 4 (df = 41) (Multiplication)
Pretest	0.29	−0.52
Posttest	5.16*	4.28*
Retention Test	4.87*	4.05*

[a] E − T difference is reflected in each *t* value.
* $p < .001$

Interview Analysis

The upper and lower thirds of each group, or a total of 16 E2, 14 T2, 16 E4, and 14 T4 students, participated in both the November and May interviews. The general patterns that emerged from the interviews are reflected in the following statements.

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1. Concerning thinking strategies actually adopted by students in the E and T groups:

   a. 80% of the E2 responses, 32% of the T2, 68% of the E4, and 40% of the T4 responses to the five facts presented reflected strategies or ways of remembering facts that were explicitly taught or encouraged during instruction. Major strategies identified within each group during the interviews include the following:

      E2: Count on (for 2 + 6); one more than a ten (for 4 + 7); one less than 10 + 6 or the +1, −1 pattern (for 9 + 6); think of a double (for 6 + 7 and 6 + 8).

      T2: Go to 10, then add on (for 9 + 6 and 6 + 8); count on. (Many children in this group tended to count on, using a ruler or fingers if necessary, for all facts. The technique was accepted by the investigator as a thinking strategy only for facts with 1 or 2 as an addend.)

      E4: Addition doubles (2 × 8); relate to time or money [5 × 6 = 30 (minutes on the clock) or 5 × 5¢ = 25¢; so 5 × 7 is 5 more (by the clock model) or 10 more (by the money model)]; figure out half, then double it (for 4 × 6, 6 × 8); patterns in the table or finger multiplication (for 7 × 9); it rhymes (6 × *8* = 4*8*).

      T4: It rhymes (6 × *4*, 2*4*; 6 × *8*, 4*8*); number your fingers or remember the patterns (for 7 × *9*).

   b. Some children either could not recall any strategy they might have used for a given fact, had simply memorized the fact, or (particularly within the T2 group), just counted on from the greater addend: 14% (E2); 55% (T2); 26% (E4); and 54% (T4).

   c. No pattern emerged from the (few) E or T group responses concerning thinking strategies for other facts.

2. Concerning thinking strategies adopted by high- and low-achieving students:

   a. The responses of students indicated that both high and low achievers learned strategies taught during the instructional periods. High-achieving students contributed 62% of the strategies tallied; low-achieving students were responsible for the remaining 38%.

   b. Retention over time (for facts not mastered during the eight-week instructional period) of strategies not reinforced by the classroom teacher appeared lower for the low group than for the high group. The 38% figure reported above reflects this loss of retention.

   c. For facts mastered during the eight-week period, however, retention of strategies used earlier was more consistent for both high and low achievers.

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Discussion

The findings from the data above are consistent with other research that supports the use of thinking strategies to facilitate the learning of basic facts. Further, E and T differences generally favoring the E groups support the overall viability of the experimental sequence and its emphasis on strategies as an approach to helping children learn the basic arithmetic facts.

The data generally yielded a consistent pattern, except for the T2 performance. Part of the fluctuation in that group's scores may be due to the immaturity of the children involved. On the addition posttest the group performed significantly better than before treatment, but on the subtraction posttest (contrary to the behavior of all other groups) a significant *loss* was recorded. The fact that both tests were administered back-to-back during the same class period seems to preclude the possibility that the children had a bad day. Still, a closer look at the addition posttest data for this group in terms of harder facts correctly answered reveals a drop from the pretest mean. A certain insecurity on the part of the students is sensed from the fact that the children seemed to elect the easier fact problems. Perhaps this same insecurity pervaded the group's efforts for subtraction. Discussions with the teacher of the group seemed to indicate that on the subtraction pretest the students freely skipped problems they did not know and accumulated a greater score on easy facts. But on the posttests and retention tests children tended to try most problems, perhaps because they felt they should know them. Within the eight-week instructional period the children had studied, though not to mastery, all the subtraction facts. The finger counting that seemed to prevail during the posttests and retention tests for subtraction took extra time and may have contributed to the depressed scores.

The learning curves for the fourth-grade groups showed great similarity. Whereas posttreatment differences favored the experimental group throughout, pretest–posttest achievement increased, and the posttest retention growth for multiplication and slight retention losses for division were nearly parallel.

Perhaps the most prominent feature of the interview data was the recurring suggestion that many children seemed to adopt strategies that were explicitly taught, encouraged, or otherwise suggested during instruction. This idea pervaded discussions with both E and T groups, at both grade levels, and with both ability groups. As was expected, the variety and number of strategies described by E group pupils was greater than that of students in the T groups. Perhaps, in view of E group performance in this and other studies, curriculum and classroom efforts should focus more carefully on the development of strategy prior to drill on basic facts.

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Secondary School Course in Applications of Mathematics to Science; M. P. Goodstein; Central Connecticut State College; 24 months; $37 200.

Experimental Career-oriented Degree Programs in the Mathematical Sciences with Emphasis on Practical Experience; J. Spanier; Claremont Graduate School and University Center; 32 months; $214 700.

Mathematics Curriculum Improvement Study; H. F. Fehr; Columbia University, Teachers College; 6 months; $3 498.

Computer-Assisted Data Analysis; M. R. Novick; University of Iowa; 12 months; $90 700.

Modules in Applied Mathematics; W. F. Lucas; Mathematical Association of America; 12 months; $4 400.

Diagnostic and Instructional Services for Undergraduate Students of Statistics; J. A. Warren; University of New Hampshire; 12 months; $182 900.

Guidebook for Use of Computer Graphics in Undergraduate Mathematics; G. J. Porter; University of Pennsylvania; 36 months; $68 200.

Development of Selected Undergraduate Course Materials in Applied Mathematical Modeling; E. J. Beltrami; State University of New York, Stony Brook; 24 months; $131 700.

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Use of Computers in the Development of Science Career Awareness in Elementary School Children; A. L. Korotkin; Richard A. Gibboney Associates; 22 months; $97 899.

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