research papers

RES01

Raising attainment in primary number sense: from counting to strategy

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Contents

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1 Summary

Raising Attainment in Primary Numeracy, a project funded by the Nuffield Foundation from 1995 to 1996, was carried out at King's College, London. It examined various teaching strategies in mathematics and their impact on the learning of low-attaining pupils in Year 3 (7- and 8-year-olds).

The research team worked with teachers and children to explore ways of improving the children's knowledge of number and how they approached calculations. The target group was made up of Year 3 children identified as under-achieving in mathematics relative to their attainment in English. The resulting changes in attainment levels were compared with those of a matched control group of pupils in similar schools.

The intervention strategies developed during the project were successful on two distinct measures:

- The targeted pupils substantially out-performed the children in the control group in terms of numbers of items correctly answered.
- The targeted pupils also demonstrated more effective strategies for answering questions correctly than the control pupils. This was demonstrated through an item-by-item analysis of the methods children used to tackle questions and find the correct answers. The teaching methods used in the research appeared to be effective in helping pupils move on from counting-based methods to more efficient mental ones.

The strategies used in the research draw attention to the need to:

- recognise mental strategies as central to becoming numerate
- acknowledge the difficulties that many pupils have in 'abstracting' mathematics from teaching activities
- build links between problem-solving concepts and number concepts
- acknowledge the importance of careful assessment of what pupils can do

2 Background to the project

In 1992, the School of Education in King's College, London, set up an initial project in collaboration with two London Local Education Authorities (LEAs) to look at ways of developing primary pupils' competence and confidence with numbers. The project was particularly interested in pupils who had been identified as low attainers at the end of Year 2.

The results of this first phase of the work included greater competence with numbers from the targeted pupils – in particular, the use of a wider range of mental and other strategies – and an increase in their confidence in tackling unfamiliar problems.

The Nuffield-funded follow-up project (the subject of this paper) was based on these observations. Its aim was to collect data on an improved model, and to use it to clarify the extent to which claims of raising standards could be made and justified.

Traditional models of remedial programmes in numeracy tend to concentrate on the inculcation of arithmetical 'facts', in the belief that a core of basic knowledge will lay the foundation for later understanding and application. The research shows this method to be inadequate (see below).

The approach we proposed is, by contrast, based on a view of numeracy as **the possession of an integrated network of knowledge, understanding, techniques, strategies and application skills** concerned with numbers, number relations and operations. Implicit in this view of numeracy is the belief that raising awareness among teachers of the nature and development of numeracy, and of the detailed achievements and difficulties of their pupils, will help them to focus the teaching they provide.

3 Theoretical underpinnings of the project

Two buses, three bicycles and four cars went past the school gate. How many wheels went by?

After agreeing that there would be six wheels on each bus, Tom and Sam, both 8 years old, quickly agreed that there would be 12 + 6 + 16 wheels. Tom counted on from 12 and announced 33 as his answer. Sam, after a few moments' reflection, announced that it was 34. Asked to explain his method, he replied "Well there's 12, and the 10 from there [pointing to the 16] makes 22; there's another 6 left [from the 16], so that's 28. Two from there [the 6] makes 30 and there's 4 left so that's 34."

Tom and Sam are friends, live near each other, play together and have been in the same class, following the same sequence of mathematics instruction, since they started school. So why is it that one has developed more efficient and flexible strategies than the other? And is it possible to help Tom develop approaches more akin to Sam's? While the former question may be impossible to answer, the latter is more amenable to systematic investigation and was the main focus of this project.

Previous research suggests that the two main aspects of mental mathematics – known facts and derived facts – are complementary. Studies of the arithmetical methods used by 7- to 12-year-olds demonstrate that higher-attaining pupils are able to use known number facts to figure out other number facts (Gray, 1991; Steffe, 1983).

For example, a child may 'know by heart' that 5 + 5 = 10 and use this to 'figure out' that 5 + 6 must be 11: that is, one more than 5 + 5. At a later stage, she or he may know by heart that $4 \times 25 = 100$ and use this fact to figure out that 40×24 must be 960.

The evidence suggests that those who can make these links between recalled and deduced number facts make good progress, because each approach supports the other. Eventually, some of the number facts the child has been deducing become 'known' number facts. As his or her range of known number facts expands, the range of strategies available for deriving new facts expands alongside it.

It is also clear, however, that there are many children – even at the end of primary school, in Year 6 – who rely more on procedures such as counting to find the answers to calculations, and do not make as much progress.

Tom and Sam represent these two extremes. On entry to Year 3, Sam has a bank of known facts and a variety of procedures to draw upon, and looks set to succeed. Tom, in contrast, is reliant upon counting and may make less progress. This project set out to explore intervention strategies to help children like Tom to develop more efficient and effective ways of deriving number facts.

4 Methods

The research sample was drawn from Year 3 classes in six primary schools. The Year 3 class teachers participated in the project, and identified eight children from each class as low attainers in mathematics. For practical purposes, 'low attainer' was defined as a 'child operating below or just into level 2' by the teacher's assessment and in the national tests at the end of Key Stage 1. The selection of low-attaining children, rather than those with special educational needs in mathematics, was intentional.

The project officer, in conjunction with class teachers, identified a similar group of pupils in six matched control schools. Thus there were 48 target pupils and 48 control pupils taking part in the project.

The six Year 3 teachers from the project schools were released one day per week for twenty weeks over the Autumn and Spring terms 1995–96. In the first term, the teachers focused on the use and interpretation of diagnostic interviews. In the mornings, they worked intensively with their group of targeted pupils in their own schools, in two sub-groups of four. In the afternoons, the teachers came together to discuss the teaching strategies being developed and to work on identifying effective intervention strategies. Research findings, such as Gray (1991), were used to inform the discussion.

In the second term, the major element of the afternoon sessions involved the teachers taking it in turns to work with a group of pupils. These sessions took place at an LEA centre, using a room with a one-way mirror to facilitate observation. The teachers watched each other teach in order to help them identify any difficulties that the children were having and to develop effective strategies to tackle them.

5 Results

5.1 Pupils' responses: quantitative results

The children's progress in quantitative terms was monitored using a framework for charting understanding and a related diagnostic interview (now published as Denvir, 2001).

The children taking part in the project and those in the control groups were assessed twice using the diagnostic interview: once near the beginning of the Autumn term 1995 and again in the Summer term 1996. Figure 1 opposite shows the mean test gains for pupils over this period.

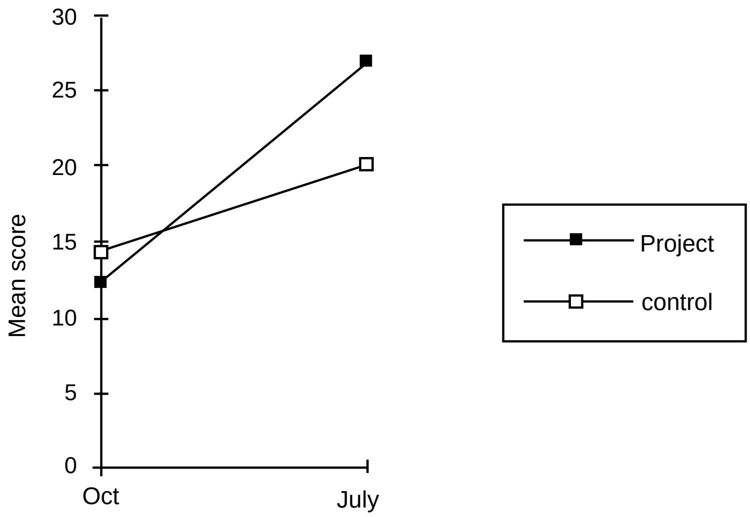


Figure 1: Mean test gains

Figure 1 clearly shows that the project pupils made greater gains than the control pupils in terms of the number of items correctly answered in the diagnostic assessment. This gain was statistically significant at the 0·05 level (that is, the likelihood of this difference coming about just by chance was 5 percent).

The assessment was designed not only to record whether or not a child could find the correct answer to a question but also the way she or he arrived at the solution. The research set about measuring changes over time in the way the children set about solving the questions in the assessment.

The methods the children chose to find a solution were coded under one of six headings, organised in increasing order of sophistication.

- Not understood (NU) A child's response was recorded as not understood if she or he could not answer the question through lack of comprehension.
- Modelling (M) This indicates that the child used physical objects, including fingers, to find the answer to the question.
- Counting (Co) This means the pupil used a counting on or counting back method, without recourse to physical objects.
- Place value (PV) Where the children used their knowledge of place value and base-10 blocks to answer a question, they were coded PV. This category was not appropriate for all questions.
- Known fact (KF) When a pupil answered too rapidly to have used a calculating strategy and indicated that she or he simply knew the answer, this was coded as a known fact.
- Derived fact (DF) This coding was used to indicate that a pupil drew on their bank of known facts to deduce another fact.

Every item on the assessment was examined for evidence of changes in strategies between the two sessions. Figure 2 overleaf shows the changes on items that the child had not understood on the first assessment. If she or he made a minor error in calculating an answer but the method was correct, then this was coded against the method used. If she or he used an inappropriate method, or was wildly incorrect, the response was coded NU.

Some items that were not understood by the children on the first assessment remained so the second time around, but the proportions for the project and control groups were very different. Nearly 70 percent of the items not understood by those in the control group in October were still not understood in July. By contrast, nearly 70 percent of the items not understood at first by those in the project groups were answered using a range of appropriate strategies. These changes are highly significant statistically (p=0·001: that is, the likelihood of this difference coming about just by chance was 0·1 percent).

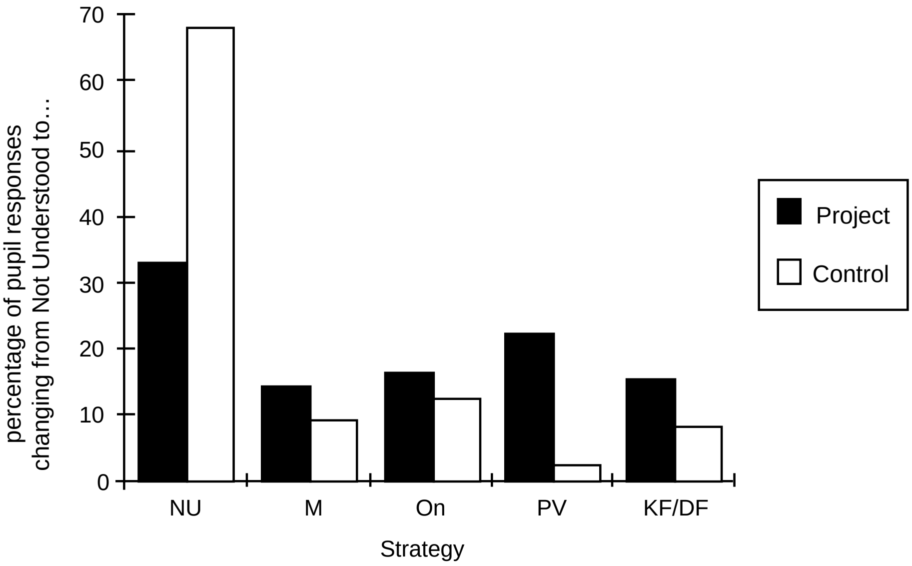


Figure 2: Changes in pupil strategies from Not Understood Oct–July

The range of strategies used by both control and project pupils on items not previously understood spanned modelling through to known and derived facts, but in every category the project pupils out-performed the control pupils.

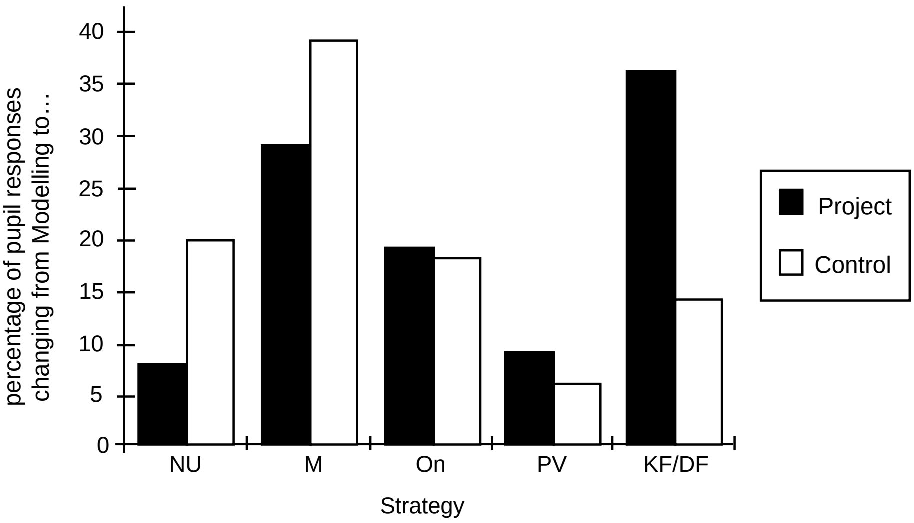


Figure 3: Changes in pupil strategies from Modelling Oct–July

Figure 3 above shows the percentage changes away from a simple modelling strategy. On several items, both groups of children continued to use modelling at the later date, and, in raw terms, the movement away from modelling is similar for both groups, with around 70 percent of project pupils and around 60 percent of control pupils using a different strategy. The main difference is that much of the movement on items for children in the control groups is accounted for by regression, with almost 20 percent of questions that had been answered using modelling the first time coded as not understood second time around. The extent of regression by the children in the project groups was markedly less, at around 8 percent. Again, these changes are highly significant (p=0·001). So again, the likelihood of this difference coming about just by chance was 0·1 percent.

Particularly striking is the change from using a modelling strategy to using known or derived facts. Thirty-six percent of the items that project pupils had originally answered using a modelling strategy were subsequently answered using a known or derived fact. The corresponding figure for control pupils was 16 percent.

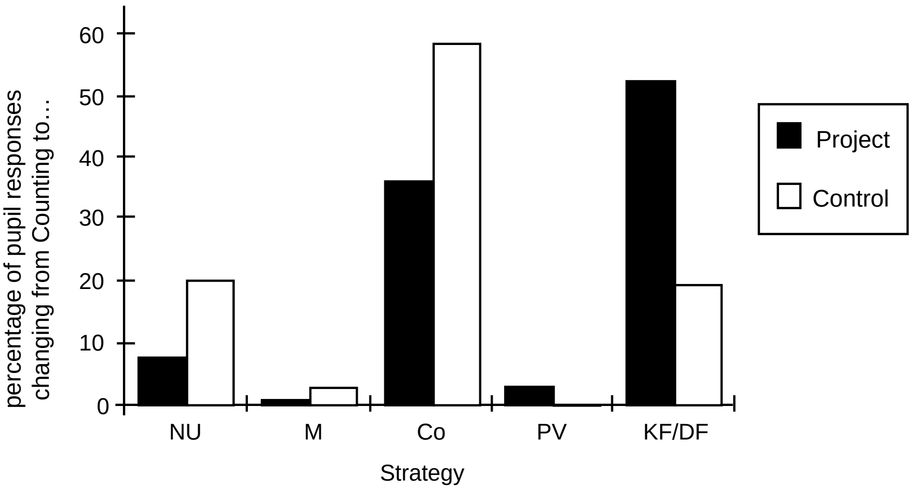


Figure 4: Changes in pupil strategies from Counting Oct–July

Figure 4 shows that at the second assessment point in both groups, there was either no change or some regression on several questions answered using a counting strategy in the first assessment. The figures for the two groups are again markedly different. The children in the control groups had made no progress in strategies used in 81 percent of the items, compared to just 45 percent of those in the project. Again, these findings are highly significant statistically (p=0·001). So again, the likelihood of this difference coming about just by chance was 0·1 percent.

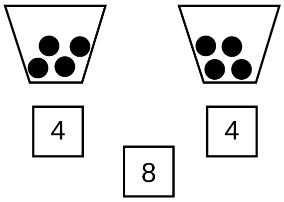
Project pupils substantially out-performed control pupils on movement from counting strategies to the use of known and derived facts, with 51 percent of items as compared to 19 percent.

All the data indicates that both in terms of the number of items correctly answered and the range of strategies used, project pupils significantly out-performed control pupils.

5.2 Pupils' responses: qualitative results

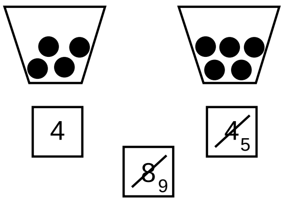
The progress of the children was also monitored through observation. This was done in normal classroom conditions and from the data gathered for the small group of pupils used in the mirror room sessions. An example from one child, Ben, illustrates how low attainment is not simply a 'problem' of the child but can be a consequence of the interaction between child, activity and teacher.

Ben knew that 4 + 4 = 8 but was unable to make the link that 4 + 5 must be 9. Every time Ben was asked to do a calculation he treated it as a new situation to be worked out afresh, so rather than using his knowledge of 4 + 4 to find the answer to 4 + 5 he chose to use a counting method. The key to solving Ben's difficulty was to get him to make some intermediate recording. He was asked to place 4 counters in each of two pots and record the situation, a known fact which he could do (below).



Ben's known fact

Ben was then told to add another counter to one of the pots and asked if the number cards were still correct. Ben not only knew that they were not but was able to 'correct' the recording to match the new situation. He could then do that without recalculating the total but by using his recording of the known fact.



Ben's derived fact

After Ben had made this connection, his teacher reported a marked change in his attitude and approach to mathematics, demonstrating an awareness that it was something he could do in his head, rather than having to rely on external counting materials.

An analysis of the transcript of an earlier session with Ben in the mirror room suggested that part of his difficulty might have arisen from trying to do what he believed the teacher expected of him, rather than attending to the mathematics. In trying to help Ben make the connection between double 4 and 4 + 5 the teacher set up a model of 4 + 4, checked that Ben knew the answer, then asked him to add another counter before asking how many there were now. The teacher was clearly using the word 'now' to try to link the two situations, but it appears that Ben interpreted the term differently. 'Now' seemed to suggest to him something on the lines of, "You have finished that one, now do this one." In other words, rather than encouraging Ben to make connections between the two calculations, the teacher had communicated the opposite. Adding 4 and 5 was a completely new task, not something that arose out of the previous one.

This is consistent with the way that most pupils meet arithmetic. Pages of 'sums' represent a random ordering of questions, each one to be answered independently of the one before. Our findings suggest that the structure and order of examples needs to be given careful attention, and that the links between them, if present, should be made explicit to the children.

Other pupils, like Ben, also seemed to be doing what they thought was expected of them, rather than relying on their mathematical understanding. For example, the teachers would often ask the children to count out, say, 10 cubes. Moments later, when they were asked how many cubes were there, the children would re-count them. In discussion, it became clear that the teachers did not discourage this. They felt it showed either that the children lacked confidence or that the children needed to reinforce their counting skills. Once the teachers started asking the children if they could remember how many there were without counting, however, they could answer easily. In re-counting, the children seemed to be responding to what they thought their teachers expected of them, rather than doing something they needed to do.

6 Teaching strategies

It was not our main intention to develop models for working with pupils on a one-to-one basis, but a pattern of working emerged that appeared to be particularly effective. The 15 minutes or so that the teachers spent working individually were split into four sections, as follows.

- Practising counting skills (2–3 minutes)

The children would work on counting on in 2s, 5s or 10s forwards and backwards from different starting numbers. They would also work on subitising skills (recognising the number of objects in small collections without counting).

- Revising individual known facts (2 minutes)

The teachers kept an envelope where they and the pupil recorded what an individual knew in number facts, and spent some time reinforcing these.

- Building on a known fact (8 minutes)

The teacher and pupil worked on deriving number facts from one of the child's known facts. This provided the main teaching emphasis for the session.

- Working with large numbers or problem solving (2 minutes)

The final minutes were spent either exploring what could be derived in terms of large numbers (for example, working on what double 400 must be if a pupil knew double 4) or putting the number facts being worked on into the context of a problem.

7 Conclusions

Statistical analysis clearly demonstrates that the intervention strategies developed were successful. They substantially increased the quantity of number questions that the targeted pupils were able to answer correctly, and significantly improved the profile of the techniques used by the pupils to arrive at correct solutions.

A primary implication of these findings is that we do not need to wait for children to be 'ready' to be taught new strategies. Through carefully targeted teaching, pupils who have not developed these strategies for themselves can indeed learn them.

The analysis of the qualitative data raises questions about the extent to which low attainment is actually the result of some 'deficit' in the child. It seems, rather, to be something that is constructed between the teacher and pupil through neither of them being totally clear about the expectations of the other. This is an important area for further research.

8 References

Denvir, H (2001) *Diagnostic assessments* BEAM Education, London

Gray, E M (1991) 'An analysis of diverging approaches to simple arithmetic: preference and its consequences' *Educational Studies in Mathematics* 22(6), 551–574

Steffe, L P (1983) 'Children's algorithms as schemes' *Educational Studies in Mathematics* 14, 109–125.

Appendix
Primary school age groups in England and Wales

key stage	***school year***	***children's age***
Foundation	early years	3–5 years
	Reception	4–5 years
Key Stage 1	Year 1	5–6 years
	Year 2	6–7 years
Key Stage 2	Year 3	7–8 years
	Year 4	8–9 years
	Year 5	9–10 years
	Year 6	10–11 years