Developing fluency in addition and subtraction facts

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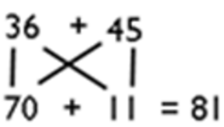
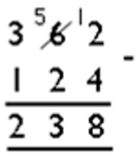
Why our focus on fluency in addition and subtraction facts?





Informal/mental addition by partitioning:

Root addition facts

3 + 4, 6 + 5, 7 + 1, 0 + 1

Formal subtraction with column method

Root subtraction facts

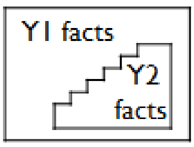
12 – 4, 5 – 2, 3 - 1

- A defined set of addition and subtraction facts builds the basis of all additive calculation, just as times tables are the building blocks for all multiplicative calculation.
- If children are not fluent in these facts, then when they are solving more complex problems the working memory is taken up by calculating basic facts, and children have less working memory to focus on solving the actual problem (See 'Is it true that some people just can't do math?' by the cognitive scientist Daniel Willingham). So fluency in basic facts allows children to tackle more complex maths more effectively.
- The importance of fluency is recognised in the national curriculum, and SATs since 2016 test children's fluency more heavily.
- Children need to be taught strategies to solve these facts. Conferencing I have done over the last few years has shown that most children don't magically become fluent in these facts even in KS2, particularly for those which bridge 10. If they aren't explicitly taught to solve e.g. 6+7 by thinking 'double 6 and one more' or to solve 12 – 8 by thinking '2 more and 2 more again' then many children will get stuck on inefficient counting based approaches.
- Counting on approaches are not only less efficient, they are associated with lower attainment in maths as well. Research by Tall and Gray found what our own extensive conferencing has shown: higher attainers tend to use known facts or derived fact strategies, and lower attainers are much more likely to use counting based approaches to solve addition and subtraction facts.

What facts have we focused on fluency in?

The full set of addition facts is here:

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10



And here are the corresponding subtraction facts.

-	0	1	2	3	4	5	6	7	8	9	10
1	1 – 0	1 – 1									
2	2 – 0	2 - 1	2 – 2								
3	3 – 0	3 - 1	3 - 2	3 – 3							
4	4 – 0	4 – 1	4 – 2	4 – 3	4 – 4						
5	5 – 0	5 - 1	5 – 2	5- 3	5 – 4	5 – 5					
6	6 – 0	6 – 1	6 – 2	6 - 3	6 – 4	6 – 5	6 – 6				
7	7 – 0	7 - 1	7 – 2	7 - 3	7 – 4	7- 5	7 – 6	7 – 7			
8	8 – 0	8 - 1	8 – 2	8 - 3	8 – 4	8 -5	8 - 6	8 - 7	8 - 8		
9	9 – 0	9 - 1	9 – 2	9 - 3	9 – 4	9 - 5	9 - 6	9 - 7	9 - 8	9 – 9	
10	10 – 0	10 - 1	10 - 2	10 - 3	10 - 4	10 - 5	10 - 6	10 - 7	10 - 8	10 - 9	10 – 10
11		11 - 1	11 - 2	11 - 3	11- 4	11 - 5	11 - 6	11 - 7	11 - 8	11 - 9	11 – 10
12			12 – 2	12 - 3	12 - 4	12 - 5	12 - 6	12 - 7	12 - 8	12 - 9	12 – 10
13				13 – 3	13 - 4	13 - 5	13 - 6	13 - 7	13 - 8	13 - 9	13 – 10
14					14 - 4	14 - 5	14 - 6	14 - 7	14 - 8	14 - 9	14 – 10
15						15 – 5	15 - 6	15 - 7	15 - 8	15 - 9	15 – 10
16							16 – 6	16 - 7	16 - 8	16 - 9	16 – 10
17								17 - 7	17 - 8	17 - 9	17 – 10

Note that in subtraction facts not all subtractions within 20 are root facts, e.g. 17-5 is not considered a root fact (7 -5 is the root fact for this).

The majority of these facts are learnt in Ys1&2.

In reception, children become fluent in working with totals to 5 (though not presented as number sentences), e.g. "Show me 5 on your hands. Now show me 5 in a different way."

Year 3 continue to focus on securing fluency in facts which bridge 10, and subtraction facts which bridge 10 in particular. Although this is a Year 2 objective, my feeling after many hours teaching and reflecting on factual fluency is that aiming for real fluency in subtraction facts such as 14 – 9 and 13 – 5 (where fluency is an answer in 3 seconds) for each and every child in Y2 is unrealistic. We feel that unless we are honest about that and accept the need to secure these facts in Year 3, we risk having children who never become secure in this.

Does fluency just mean memorisation?

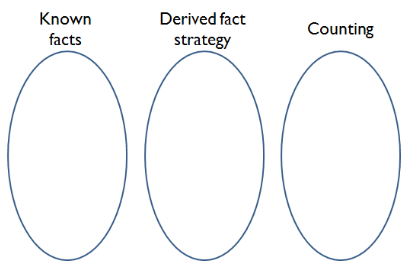
Not necessarily – when you conference adults on how they solve addition and subtraction facts, almost all adults rely on very quick use of strategies to solve some of them. Reflect carefully on the set of addition and subtraction facts shown: which have you memorised and which are you very quickly deriving? We've taken fluency to mean 'getting an answer pretty quickly and with limited demands on working memory', and aim for an average of 3 seconds or less per fact. My work conferencing fluent children in KS2 who were working at this speed showed:

- Most facts which didn't bridge 10 were memorised – the children reported 'just knowing' than 4 + 5 = 9 or 2 + 6 = 8 for example.
- For facts which bridge 10 the picture is more complex, and many of the facts which bridge 10 were quickly derived using strategies (but still in less than 3 seconds!).
    - Double 6, 7. 8 and 9 were always memorised in fluent children
    - Many fluent children also reported 'just knowing' that 9 + 3 = 12 and 8 + 4 = 12 and related this to their times table/skip counting knowledge.
    - Fluent children in the year groups conferenced (up to Year 4) generally reported using strategies for many of the other facts. 8 + 9 is an example of a fact that actually very few people (either adults or children) have memorised. Of the many hundreds of teachers I have asked, only about 5% report 'just knowing' that 8 + 9 = 17. Most fluent people solve this through very quickly applying a strategy: bridging through ten, near doubles or compensating (adding 10 and subtracting 1).

As a reference point, the grid to the right is a good example of the approaches taken by a fluent, high attaining Year 4 child to each of the addition facts: he doesn't use a counting approach for any of the facts, but he has certainly not memorised them all either (K= Known fact; S= Strategy)

	0	1	2	3	4	5	6	7	8	9	10
0	K	K	K	K	K	K	K	K	K	K	K
1	K	K	K	K	K	K	K	K	K	K	K
2	K	K	K	K	K	K	K	K	K	K	K
3	K	K	K	K	K	K	K	K	S	S	K
4	K	K	K	K	K	K	K	S	K	S	K
5	K	K	K	K	K	K	S	S	S	S	K
6	K	K	K	K	K	S	K	S	S	S	K
7	K	K	K	K	S	S	S	K	S	S	K
8	K	K	K	S	K	S	S	S	K	S	K
9	K	K	K	S	S	S	S	S	S	K	K
10	K	K	K	K	K	K	K	K	K	K	K

Why not try this on some of your children? Find out how they solve each of the 121 facts. They are all written out on the final page of this memo – just print and cut them out. I use these sorting circles with the children, and have found they very quickly get the idea once I have given them an example of a known fact (the vast majority of KS1 &v2 children will just know that 5 + 5 = 10 for example), a strategy fact (e.g. calculating 6 + 5 by relating it to 5 + 5) and a counting based approach. If children say they would use counting don't bother getting them to solve the fact – you will be there all day. If they say strategy I find it interesting & helpful to ask what strategy they have used: my



notes on the child whose grid is shown above for example showed that he added numbers with a difference of 2 by relating to doubling the number in between (e.g. 6 + 8 = 7 +7; 5 + 7 = 6 + 6) . It takes about 15 minutes to conference one child, and is time very well spent.

How do children become fluent?

As mentioned above, children need to be **TAUGHT** strategies to derive the facts! An interesting piece of research (Thornton, 1978) showed that teaching *strategies* is more effective in securing fluency in addition and subtraction facts than taking a rote memorisation approach. That is to say, even if your aim is memorisation, the most effective way to get there is through the teaching of strategies. There is a huge amount to unpick in this and you need to consider how children are going to become fluent in each and every fact. For example, we want children to just know that 4 + 2 = 6 and 9 – 2 = 7 etc so we need to teach children that when we add 2 or subtract 2 we are moving to the next/previous even number (if starting on an even) or odd number (if starting on an odd). Without being taught this, many children will count. e.g. for 9 – 2, "nine, eight, **seven"**. Being able to do this requires being able to count fluently in odd numbers (as well as in the more commonly practiced even numbers), something we realised we were not teaching our children to do previously.

We mapped out a teaching progression so we could identify when every individual fact was being taught, and discussed and agreed teaching approaches for each of these fact groups. In Year 1 we teach strategies for facts within 10 (steps 1 – 7) and in Year 2 we teach the bridging ten facts (steps 8 – 11).

1. Adding 1 (e.g. 7 + 1 *and* 1 + 7)
2. Doubles and near double of numbers to 5 (e.g. 3 + 3, 4 + 5, 5 + 4)
3. Adding 2 (e.g. 4 + 2 *and* 2 + 4)
4. Number bonds to 10 (e.g. 8 + 2 *and* 2 + 8)
5. Adding 0 to a number (e.g. 3 + 0 *and* 0 + 3)
6. Adding 10 to a number (e.g. 5 + 10 *and* 10 + 5)
7. The ones without a family 5 + 3, 3 + 5, 6 + 3, 3 + 6 (these pairs of facts are the only ones which don't fit in any of the other families, though the last two can be related to counting in 3s)
8. Doubles of numbers to 10 (e.g. 7 + 7)
9. Near doubles (e.g. 5 + 6 *and* 6 + 5)
10. Bridging (e.g. 8 + 4 *and* 4 + 8)
11. Compensating

Note that these 3 strategies can often be used interchangeably, e.g. for 8 + 9, some people will use near doubles (e.g. 8 + 8 + 1), some will use bridging (e.g. 8 + 2 + 7) and some will use compensating (8 + 10 – 1)

N.B. Before the children are ready to learn bridging as a strategy, they need to be able to partition all single digit numbers._Adding 8 + 5, for example, by bridging through 10 requires children to partition 5 into 2 and 3. We do an enormous amount on partitioning single digit numbers all through Year 1.

There is even more pedagogy involved in supporting children to become fluent in subtraction facts then there is in addition facts, and we've developed separate teaching guidance for our staff on that.

Once children have been taught the strategies, they need to move on to **PRACTICE** of the facts, Remember for many facts the ultimate aim of the practice is memorisation, while for others the aim of the practice is increasing speed and fluency in the applied strategy. We have a software package which children use to practice which encourages them to aim for an average of 3 seconds or less per fact. This means that in 2 minute practice session the children should be recalling at least 40 facts: the more you practice the quicker they get, and the quicker they get, the less time it takes out of the lesson.

Generally for practice we focus on:

- Practising the set of facts being learnt (or just learnt) in isolation for a few days
- Mixing these up with all previously learnt facts

We use a mixture of a software package, practice sheets in class and flash cards/smartboards to give the children practice, in fact very similar approaches to those we take in phonics sessions. In fact we've found thinking about the structured and systematic approach we take to the teaching of phonics in general is a good analogy for thinking about structured and systematic teaching and learning of these strategies and facts.

Although working on securing fluency in these sounds on one level basic and dry, we have found children really enjoy both the discussion and reasoning that the learning of strategies involves, and the confidence they get from having these building blocks in place. In one memorable conferencing session with a middle attaining Year 4 child at a colleague's school, who had been part of a class revisiting the teaching of addition facts for a month, I asked how she would work out 6 + 9. "I'd probably add one to make ten, then add the other five to make 15," she said before continuing, "but of course the other way you could think about it is that 6 is two groups of three and 9 is three groups of three, so if you add them together you get five groups of three which is 15." This was a child who just 4 weeks before had been reliant on counting based strategies and had solved 6 + 9 by putting "nine in my head and counting on." It was really exciting to see she had gained not just greater confidence in basic arithmetic, but had come to see patterns and connections with a much much wider reach than this.